(𝒞) est une ellipse $e = \frac{c}{a} = \frac{3}{5}$

2) on pose $r = OM$; $M(r.\cos a\ ,\ r.\sin a)$

$M \in (\mathcal{C}) \Rightarrow 25(r^2.cos^2a + r^2.sin^2a) = (3r.\cos a - 16)^2$

$\Rightarrow 25\,r^2 = (16 - 3\,r\,.\cos a)^2$

$$\Rightarrow 5\,r = |16 - 3r.\cos a| \Rightarrow \begin{cases} 5r = 16 - 3r.\cos a \\ ou \\ 5r = -16 + 3r.\cos a \end{cases} \Rightarrow \begin{cases} r = \frac{16}{5+3\cos a} \\ ou \\ r = \frac{-16}{5-3\cos a} \end{cases} < 0$$

D'où

$$\boxed{r = OM = \frac{16}{5 + 3\cos a}}$$

3)a) $(OM) \cap (\mathcal{C}) = \{M', M\}$

Soit $OM' = r'$; $M'(r'\cos(a+\pi)\ ,r'.\sin(a+\pi))$

D'après 2)

$$r' = OM' = \frac{16}{5 + 3\cos(a+\pi)}$$

$$r' = OM' = \frac{16}{5 - 3\cos a}$$

$$\frac{1}{r} + \frac{1}{r'} = \frac{5 + 3\cos a}{16} + \frac{5 - 3\cos a}{16}$$

$$\boxed{\frac{1}{r} + \frac{1}{r'} = \frac{10}{16} = \frac{5}{8}}$$

b) $(\mathcal{C}): \frac{(x+3)^2}{25} + \frac{y^2}{16} = 1$

$w(-3\,,0)$ $\quad (\mathcal{C}): \frac{X^2}{25} + \frac{Y^2}{16} = 1 \quad dans\ R' = (w, \vec{i}, \vec{j})$

$D: X = \frac{25}{3}\ \ dans\ R'$ $\qquad D: x = \frac{16}{3}\ \ dans\ R = (o, \vec{i}, \vec{j})$

$$\cos(a) = \frac{\frac{8}{3}}{\overline{OI}} \Rightarrow \overline{OI} = \frac{8}{3\cos(a)}$$

$$\frac{1}{\overline{OM}} + \frac{1}{\overline{OM'}} = \frac{5 + 3\cos a - 5 + 3\cos a}{16} \quad car\ O \in [MM']$$

$$= \frac{3\cos a}{8} = \frac{1}{\overline{OI}}$$

Qarray.tn -Aqra Blech- Qarray.tn